

Axion Dark Matter with High-Scale Inflation

Eung Jin Chun

Korea Institute for Advanced Study, Seoul 130-722, Korea

We show that supersymmetric axion models breaking the PQ symmetry by the interplay of non-renormalizable supersymmetric terms and soft supersymmetry breaking terms provide a natural framework not only for generating the axion scale from soft supersymmetry breaking scale $m_{3/2}$ but also for enhancing it during inflation by factor of order $\sqrt{H_I/m_{3/2}}$ where $H_I \simeq 10^{14}$ GeV according to the recent BICEP2 result. In this scheme, the PQ symmetry can stay broken throughout the whole history of the Universe if the reheating temperature is below 10^{10} GeV, or $m_{3/2}$ when the PQ fields couple strongly to thermal (Standard Model) particles. It is also shown that parametric resonance during preheating is not effective enough to induce non-thermal PQ symmetry restoration. As a consequence, axion models with the QCD anomaly $N_{DW} > 1$ can be made free from the domain wall problem while the axion isocurvature perturbation is suppressed sufficiently for the axion scale during inflation larger than about $M_P(\Omega_a h^2/0.12)^{1/2}(F_a/10^{12}\text{GeV})^{0.6}$ GeV.

Introduction: The recent measurement of CMB B-mode polarizations by BICEP2 points to large primordial tensor perturbations with the tensor-to-scalar ratio $r = \mathcal{O}(0.1)$ [1]. This reveals a high-scale inflation with the Hubble parameter,

$$H_I \simeq 10^{14} \text{ GeV} \left(\frac{r}{0.16} \right)^{1/2} \quad (1)$$

which corresponds to the inflation energy scale of $V_I^{1/4} \equiv (3H_I^2 M_P^2)^{1/4} \simeq 2.7 \times 10^{16} (r/0.16)^{1/4}$ GeV. Furthermore, the chaotic inflation with a quadratic potential [2] seems to fit nicely other CMB observables measured by Planck [3]. If confirmed, it has profound implications for the axion dark matter [4–7]. In this work, we present a natural framework where the PQ symmetry is never restored throughout the whole history of the Universe, and the PQ symmetry breaking scale during inflation is much larger than the present one so that the domain wall problem occurring in axion models with the QCD anomaly $N_{DW} > 1$ can be avoided, and the axion isocurvature density perturbation can be sufficiently suppressed.

Axion and strong CP problem: The axion was introduced to resolve the strong CP problem in a dynamical way (for a recent review, see, [8, 9]). Being a pseudo-Goldstone boson of a QCD-anomalous $U(1)$ symmetry (PQ symmetry) the axion couples to the CP-odd QCD field strength term;

$$\mathcal{L}_{QCD} = \frac{a}{F_a} \frac{g_s^2}{32\pi^2} G_{\mu\nu}^a \tilde{G}^{\mu\nu}_a \quad (2)$$

where F_a is the axion decay constant and $\tilde{G}^{\mu\nu}_a = \frac{1}{2}\epsilon^{\mu\nu\rho\sigma} G_{\rho\sigma}^a$. That is, the QCD θ angle is replaced by a dynamical field a : $\theta \equiv a/F_a$. After the QCD phase transition, instanton effect generates the axion potential,

$$V(a) = m_a^2 F_a^2 \left[1 - \cos \frac{a}{F_a} \right] \quad (3)$$

where the axion mass is given by $m_a = \sqrt{Z}/(1+Z)(f_\pi m_\pi/F_a)$ with the the up and down quark mass ratio $Z = m_u/m_d$, and m_π (f_π) is the pion mass (decay constant). The axion potential sets the vacuum expectation value $\langle a \rangle = 0$ ensuring no θ contribution to the neutron electric dipole moment.

Axion cold dark matter: The occurrence of the axion potential (3) also plays an important role in cosmic axion production. If the PQ symmetry is broken after inflation, there appear three sources of axion production: coherent oscillation from initial misalignment by θ_i , axionic string formation upon the PQ symmetry breaking, and domain wall production during the QCD phase transition. Summing these up one gets [6, 9]

$$\Omega_a h^2 \approx 0.18 \langle \theta_i^2 \rangle \alpha_{t.d.} \left(\frac{F_a}{10^{12} \text{ GeV}} \right)^{1.19} \left(\frac{\Lambda}{400 \text{ MeV}} \right) \quad (4)$$

where $\langle \theta_i^2 \rangle \approx 2\pi^2/3$ is a value averaged over possible ranges of the initial misalignment angle θ_i which is not uniform in our Hubble volume, and $\alpha_{t.d.}$ takes into account contributions from strings and domain walls. One then finds that

the axion cold dark matter (CDM) satisfies $\Omega_a h^2 \simeq 0.12$ for $F_a \approx 1.4 \times 10^{11} \alpha_{t.d.}^{-0.84}$ GeV (see [6] and references therein for different estimates of $\alpha_{t.d.}$).

Note that the θ angle defined in (2) by $\theta \equiv a/F_a$ has a periodicity $\theta \equiv \theta + 2\pi N_{DW}$ where N_{DW} is the QCD anomaly, and thus there appear distinct N_{DW} θ vacua: $0, 2\pi, \dots, 2\pi(N_{DW} - 1)$ in the axion potential (3). It is then required to have $N_{DW} = 1$ in order not to produce stable string-domain wall networks which overclose the Universe. The KSVZ model with one pair of heavy quarks has $N_{DW} = 1$ while the original DFSZ model has $N_{DW} = 6$ [8]. Let us remark that various hybrid models can be constructed to obtain $N_{DW} = 1$.

If the PQ symmetry is broken before or during inflation, axionic strings formed during the PQ phase transition are efficiently diluted away and no domain wall can form. Thus, any axion model with $N_{DW} > 1$ is allowed, and the axion dark matter can be produced from the coherent oscillation as well as from (massless) axion fluctuations during inflation, $\delta a \approx H_I/2\pi$. This leads to

$$\Omega_a h^2 \approx 0.18 \left[\theta_i^2 + \left(\frac{H_I}{2\pi F_I} \right)^2 \right] \left(\frac{F_a}{10^{12} \text{ GeV}} \right)^{1.19} \left(\frac{\Lambda}{400 \text{ MeV}} \right). \quad (5)$$

Here a uniform initial misalignment angle θ_i is taken as our Universe might be expanded from a small patch with given θ_i during inflation, and the axion decay constant during inflation F_I is assumed to be different from the present value F_a and thus could suppress the fluctuation contribution to evade the problem with axion isocurvature perturbation [10, 11].

The axion fluctuations produced during inflation become isocurvature density perturbations of the axion dark matter after the axion acquires mass at the QCD phase transition. The recent Planck measurements constrain the isocurvature power spectrum \mathcal{P}_a of the axion CDM compared to the scalar power spectrum $\mathcal{P}_{\mathcal{R}}$ [3]:

$$\mathcal{P}_a \equiv 4\xi^2 \frac{(H_I/2\pi)^2}{(F_I\theta_i)^2 + (H_I/2\pi)^2} \lesssim 0.04 \mathcal{P}_{\mathcal{R}} \quad (6)$$

where $\xi \equiv \Omega_a/\Omega_{CDM}$, and $\mathcal{P}_{\mathcal{R}} \simeq 2.2 \times 10^{-9}$. This essentially rules out the axion as a CDM candidate [5, 6] if $F_I = F_a$. In order to see how much the severe constraint from the isocurvature perturbations can be relaxed for $F_I \gg F_a$, let us first consider two regions separately: (i) $F_I\theta_i < H_I/2\pi$, and (ii) $F_I\theta_i > H_I/2\pi$. In the case (i) having $\mathcal{P}_a \approx 4\xi^2$, Eqs. (5) and (6) require $\xi < 4.7 \times 10^{-6}$ together with

$$F_I \gtrsim 9 \times 10^{15} \left(\frac{F_a}{10^{12} \text{ GeV}} \right)^{0.6} \text{ GeV}, \quad (7)$$

and $\theta_i \lesssim 0.0018 \left(\frac{10^{12} \text{ GeV}}{F_a} \right)^{0.6}.$

In the case (ii), one finds the requirements:

$$F_I \gtrsim 4.2 \times 10^{18} \sqrt{\xi} \left(\frac{F_a}{10^{12} \text{ GeV}} \right)^{0.6} \text{ GeV}, \quad (8)$$

and $\theta_i \approx 0.82 \sqrt{\xi} \left(\frac{10^{12} \text{ GeV}}{F_a} \right)^{0.6}.$

Thus, the axion CDM with $\xi = 1$ can be obtained for $F_a \sim 10^{12}$ GeV and $\theta_i \sim 1$ if $F_I \sim M_P$ is allowed.

PQ symmetry and supersymmetry breaking: The axion decay constant changing with time is a generic feature as far as there is no particular reason to forbid inflaton coupling to the PQ fields. A simple and systematic framework can be obtained in the context of supersymmetry which has to be broken by the inflaton potential of order V_I during inflation [12]. Furthermore, the PQ symmetry breaking scale can be correlated to the supersymmetry breaking scale in a flat potential model with the superpotential [13, 14]:

$$W_{PQ} = \lambda \frac{P^{n+2} Q}{M_P^n} \quad (9)$$

where n is a positive integer, and the $U(1)_{PQ}$ charge 1 and $-(n+2)$ are assigned to P and Q , respectively. Recall that the dynamical generation of the axion scale from soft supersymmetry breaking terms can also resolve the μ

problem by introducing a Higgs bilinear operator $W_{KN} = hP^{n+1}H_uH_d/M_P^n$ [15] which ensures $\mu \sim m_{3/2}$ and realizes a supersymmetric version of the DFSZ axion model having $N_{DW} = 6$. The potential domain wall problem is evaded if the PQ symmetry is broken before/during inflation and remains broken during the whole history of the Universe as will be discussed below.

In order to deliver the essential features, let us consider a simplified PQ superpotential together with the quadratic potential for chaotic inflation¹:

$$W = m_\chi \chi^2 + \frac{\lambda}{n+3} \frac{\phi^{n+3}}{M_P^n} \quad (10)$$

where χ is the inflaton superfield and ϕ is a representative of the PQ superfields. As is well-known [12], a non-renormalizable coupling allowed in the Kahler potential

$$\delta K \sim \frac{1}{M_P^2} \chi^\dagger \chi \phi^\dagger \phi \quad (11)$$

delivers the supersymmetry breaking effect by the inflaton to the PQ sector through F-terms leading to the scalar potential $V = V_{soft} + V_{SUSY}$:

$$V_{soft} = (C_m H^2 + m_\phi^2) |\phi|^2 + \left[\frac{(C_A H + A) \lambda}{n+3} \frac{\phi^{n+3}}{M_P^n} + h.c. \right],$$

$$V_{SUSY} = |\lambda|^2 \frac{|\phi|^{2n+4}}{M_P^{2n}} \quad (12)$$

where C_m and C_A are constants of order one, and m_ϕ and A are the usual soft terms of order $m_{3/2}$. Minimization of the above scalar potential gives rise to the vacuum expectation value $\langle \phi \rangle = \phi_0$ as follows [17]:

$$\phi_0 \sim \begin{cases} (H_I M_P^n)^{\frac{1}{n+1}} \sim F_I & \text{during inflation} \\ (\text{Min}[H, m_\phi] M_P^n)^{\frac{1}{n+1}} & \text{after inflation} \\ (m_\phi M_P^n)^{\frac{1}{n+1}} \sim F_a & \text{after reheating.} \end{cases} \quad (13)$$

One can see that the PQ symmetry remains broken during the whole history of the Universe if the reheating occurs at the Hubble parameter smaller than m_ϕ , which sets an upper limit on the reheat temperature:

$$T_R \lesssim 10^{10} \left(\frac{m_\phi}{200 \text{ GeV}} \right)^{1/2} \text{ GeV}. \quad (14)$$

to maintain the initial PQ symmetry breaking vacuum stays connected to the present one. The reheat temperature needs to be even lower if a PQ field has a sizable coupling to any fields in thermal equilibrium and thus obtains a thermal mass. For instance, the PQ field may couple to a right-handed neutrino, $W \sim y \phi N N$ to realize the seesaw mechanism [13, 14], or to a heavy quark, $W \sim y P Q Q^c$ in the KSVZ model. These couplings generate thermal mass-squared of order $\delta m_\phi^2 \sim y^2 T_R^2$ which restores the PQ symmetry if $\delta m_\phi^2 > m_\phi^2$. To avoid this, we put a rough condition of

$$T_R \lesssim \frac{m_\phi}{y}. \quad (15)$$

Another important issue concerning symmetry non-restoration is parametric resonance which can leads to huge production of bosonic fluctuations in the process of preheating during the inflaton oscillation period, and thus non-thermal symmetry restoration [18, 19]. If this happens, subsequent symmetry breaking brings back the topological defect production and thus the results in (4) should be applied for $N_{DW} = 1$ while ruling out axion models with $N_{DW} > 1$. This is a generic feature if the PQ field has a direct coupling to the inflaton: $\delta \mathcal{L} \sim g^2 |\chi|^2 |\phi|^2$. Even without such a coupling, the PQ symmetry can be restored if there is a large initial value ϕ_i during the inflation oscillation period before the reheating, and PQ symmetry non-restoration is ensured if $F_a \gtrsim 10^{-4} \phi_i$ [20]. In supersymmetric theories, there appear couplings between the inflaton and PQ fields through the supersymmetry breaking effect (12)

¹ Note that a special form of the Kahler potential is required to guarantee the quadratic term in the scalar potential: $V_I = m^2 |\chi|^2$ [16].

although direct couplings in the superpotential are forbidden. In this case, parametric resonance is never effective as we will see in the following. To simplify the analysis, let us take $n = 2$ and ignore the A terms ($m_\phi \gg A$ and $C_m \gg C_A$), and assume negative mass-squared terms²:

$$V = -(C_m H^2 + m_\phi^2)|\phi|^2 + \frac{\lambda^2}{M_P^2}|\phi|^6. \quad (16)$$

When the mass term is constant (during inflation and $H \ll m_\phi$), the homogeneous field value $\phi_0 = \langle \phi \rangle$ is set by $\phi_0 = \sqrt{C_m H_I M_P / \sqrt{3}\lambda}$ during inflation, and $\phi_0 = \sqrt{m_\phi M_P / \sqrt{3}\lambda}$ after reheating for $H \ll m_\phi$. During the period of inflaton oscillation before reheating ϕ_0 follows the equation of motion

$$\ddot{\phi}_0 + 3H\dot{\phi}_0 + \frac{3\lambda^2}{M_P^2}\phi_0^5 - C_m H^2 \phi_0 = 0 \quad (17)$$

ignoring the m_ϕ^2 term. It is solved by

$$\phi_0 = A \sqrt{\frac{H M_P}{\sqrt{3}\lambda}} \quad (18)$$

where $H = 2/3t$ and the coefficient A is determined to be $A = \left(\frac{4}{9}C_m + \frac{1}{4}\right)^{1/4}$ which is slightly different from the ones during inflation and after reheating. Treating properly the transition between the inflation (reheating) and oscillation periods, one would be able to find a smooth transition of $A(t)$ connecting the different values during inflation, inflaton oscillation, and radiation-dominated periods. For our analysis, we will take a simple approximation of an abrupt change of periods and use the solution (18) to find the evolution of fluctuations of the PQ field: $\phi = \phi_0 + \delta\phi$, during the oscillation period. The linearized equation of motion for the fluctuations in Fourier space is

$$\ddot{\delta\phi}_k + 3H\dot{\delta\phi}_k - \left(\frac{k^2}{a^2} + \tilde{C}_m H^2\right)\delta\phi_k = 0 \quad (19)$$

where $\tilde{C}_m = \frac{11}{9}C_m + \frac{5}{4}$. Note that the source term does not contain the oscillating term of the inflaton contrary to the usual parametric resonance case with a direct coupling between the inflation and PQ fields in non-supersymmetric models [18–20]. Therefore, no parametric resonance can occur and it should remain true even after considering the transition effect properly. One can indeed find that an asymptotic behavior of solutions to the equation (19) is given by

$$\delta\phi_k \sim \frac{\sin(kt^{3/2})}{k^{4/5}t^{4/3}} \quad (20)$$

which is an oscillating and decaying function showing no resonance effect.

Let us finally see how large $F_a \gtrsim 10^{12}$ GeV with $F_I \gtrsim M_P$ in (8) can be allowed in our framework. As we have $F_I/F_a = \sqrt{C_m H_I / m_\phi}$ independently of n , the first relation of (8) translates to

$$C_m \gtrsim 3\xi \left(\frac{m_\phi}{10^2 \text{GeV}}\right) \left(\frac{10^{13} \text{GeV}}{F_a}\right)^{0.8} \quad (21)$$

and $F_a \sim 10^{13}$ GeV requires a small Yukawa coupling of $\lambda \sim 10^{-6}$ for $n = 2$. Note that one may have $F_a \gg 10^{12}$ GeV allowing trans-Planckian PQ field values during inflation, $F_I \gg M_P$, which, however, does not lead to a dangerous effect in the potential as it comes with a small Yukawa λ .

Conclusion: The QCD axion is a well-motivated hypothetical particle which is a pseudo-Goldstone boson of the PQ symmetry solving the strong CP problem and is a good candidate for the cold dark matter. In generic axion models with the QCD anomaly $N_{DW} > 1$, the PQ symmetry needs to be broken before/during inflation and never

² For a detailed analysis with general soft terms for the original superpotential (9), see Ref. [21].

restored after inflation in order to prohibit production of stable domain walls. However, the isocurvature perturbation produced during inflation rules out axions as a dark matter candidate if the PQ symmetry breaking scale F_I during inflation is the same as the present one F_a . In general, these two symmetry breaking scales need not to coincide, for instance, if couplings between inflation and PQ fields are allowed. Supersymmetry provides a natural way to realize such a situation. Generic order-one couplings between inflaton and PQ fields allowed in the Kahler potential lead to mediation of the inflaton supersymmetry breaking effect to the PQ sector through F-terms. A smooth connection of such an early supersymmetry breaking to the present one can be realized if the PQ symmetry breaking is induced by soft supersymmetry breaking terms as presented in this work. As a consequence, the stringent constraint from isocurvature perturbations can be avoided if $F_I \gtrsim M_P(\Omega_a h^2/0.12)^{1/2}(F_a/10^{12}\text{GeV})^{0.6}$.

For this to happen, the reheat temperature has to be low enough; $T_R \lesssim 10^{10}$ GeV in order to keep the Hubble parameter below the soft supersymmetry breaking scale $m_{3/2}$. If PQ fields couple strongly to thermal particles through a coupling y , it is required to have $T_R \lesssim m_{3/2}/y$ in order to forbid the induced thermal masses which can restore the PQ symmetry. There could also be non-thermal symmetry restoration due to parametric resonance during the inflaton oscillation period. Having the inflaton coupling to the PQ fields only through the Hubble parameter, the inflaton supersymmetry breaking effect, parametric resonance is shown to be ineffective in our framework.

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